The Cavendish Experiment

Toki Nishikawa, Hannah Magoon

February 6th, 2023

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1 Introduction

In 1687, Newton proposed his law of universal gravitation, which states that two spherical objects with masses m and M are mutually attracted by a force F that acts along the line connecting their centers. This law is described by the following equation

$$F = G\frac{mM}{r^2} \tag{1}$$

where r represents the distance between the centers of the spheres and G is a universal constant. Obtaining a value for G experimentally can be quite challenging as it requires accurately measuring the force of attraction between two known masses, and in a lab setting, these forces are minuscule.

Henry Cavendish achieved the first measurement of G in 1797 using a torsional pendulum similar to the one we will use in our experiment. As shown in **Figure 1**, two small spheres of equal mass, A and B, were placed on opposite ends of a beam, which was suspended from its center, C, by a wire. Two larger spheres of equal mass, P and Q, were positioned in the horizontal plane as indicated by the illustration.

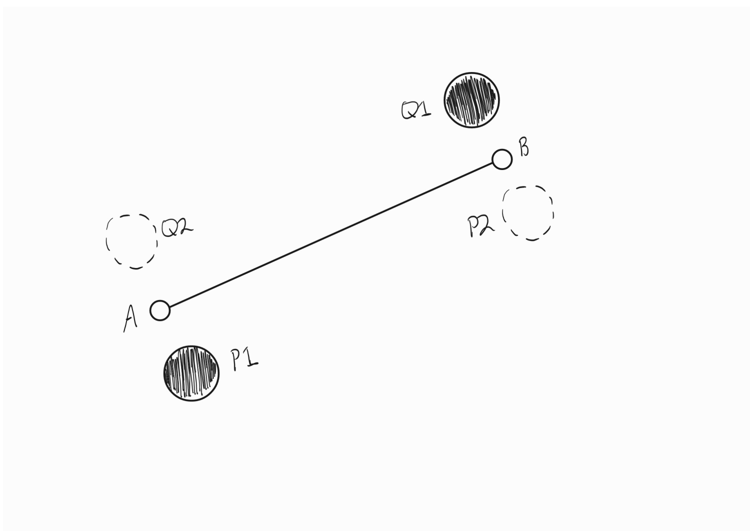

Figure 1: The Cavendish balance

The gravitational attraction between each smaller sphere and its adjacent larger sphere created a torque around point C, causing a twist in the wire. Cavendish accurately measured the magnitude of the twist by reflecting light off a mirror attached to the beam and observing the position of the reflected light on the wall.

The wire and the beam form a torsional pendulum. In the absence of external torque, the pendulum will unwind to an equilibrium orientation θ_0, where the wire experiences no internal rotational stress. With that said, external disturbances are unavoidable, and the pendulum will never be exactly stationary. A small perturbation from equilibrium will exert a restoring torque τ given by the equation

$$\tau = -\beta(\theta - \theta_0) \tag{2}$$

where β is a constant determined by the properties of the wire. It is evident from **Equation 2** that τ is proportional to $\Delta\theta$, indicating that the period T is independent of the pendulum's amplitude.

In the presence of an external torque τ_{ext}, the torsional pendulum will find a new equilibrium position θ at which the total torque is now zero, or

$$\tau_{ext} - \beta(\theta - \theta_0) = 0 \tag{3}$$

To calculate the strength of the gravitational force from this apparatus, Cavendish needed to know the torsion constant β. However, he was unable to identify β through calibration with a known torque since the wire had too small a torsional strength. Instead, Cavendish determined β by timing the oscillations due to random external disturbances.

2 Experimental Setup

We use a compact version of the Cavendish balance from TEL-Atomic, shown in **Figure 2**, to measure the gravitational constant. This torsional pendulum is an aluminum beam with two small lead spheres attached to either end. The beam is suspended at its center by a fine tungsten wire with a diameter of $25\mu m$ and can rotate horizontally. To prevent any disturbances from air currents, the pendulum is enclosed between two transparent walls. Outside the walls, a second beam with two large tungsten spheres is free to rotate in order to alter the torques acting on the inner beam.

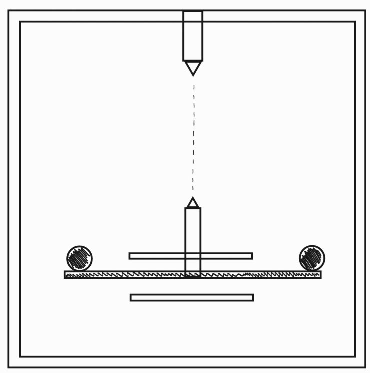

Figure 2: The TEL-Atomic pendulum

To determine the rotation angle of the beam, we will reflect a laser off of a mirror attached to the support for the beam. The experiment has been set up so that the long axis of the inner beam is parallel to the face of the mirror and the transparent plates. By tracking the position of the reflected laser on a ruler, we can measure the changes in angular position of the inner beam. As we vary the position of the outer beam, the torque applied to inner beam will change, altering the equilibrium orientation of the pendulum.

3 Uncertainty

If the variables **x** are independent, the error propogation law becomes

$$\sigma^2(y_i) = \sum_{j=1}^{n} (\frac{\partial y_i}{\partial x_j})^2 \sigma^2(x_j) \tag{4}$$

In **Section 5.1**, we will need to calculate the uncertainty of the mean y_0 value for each orientation. The equation for the mean is

$$\bar{x} = \frac{x_1 + x_2 + ...}{n} \tag{5}$$

Then, according to the error propagation rule,

$$\sigma(\bar{x}) = \sqrt{(\frac{1}{n})^2 \sigma^2(x_1) + (\frac{1}{n})^2 \sigma^2(x_2) + ...} = \frac{1}{n}\sqrt{\sigma^2(x_1) + \sigma^2(x_2) + ...} \tag{6}$$

In the same section, we will also need to calculate the uncertainty of θ, given by

$$\theta = \frac{1}{2} tan^{-1}(\frac{A}{B}) \tag{7}$$

According to the error propagation rule,

$$\sigma(\theta) = \sqrt{[\frac{B}{2(A^2+B^2)}]^2 \sigma^2(A) + [-\frac{A}{2(A^2+B^2)}]^2 \sigma^2(B)} \tag{8}$$

4 Results

Our results are summarized in **Figures 3-8**. We used the least squares method to find initial values for our parameters. These initial parameters are summarized in **Table 1**.

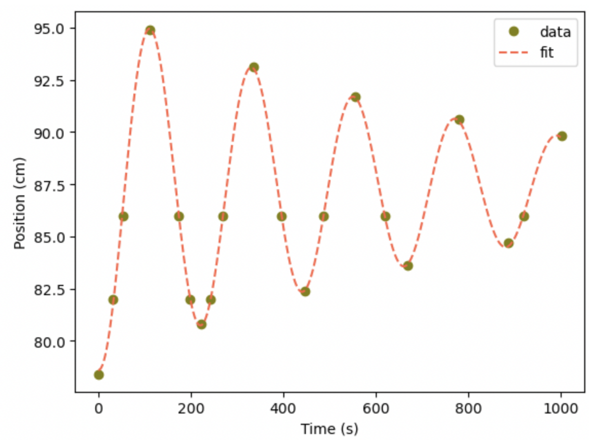

Figure 3: First orientation, trial 1

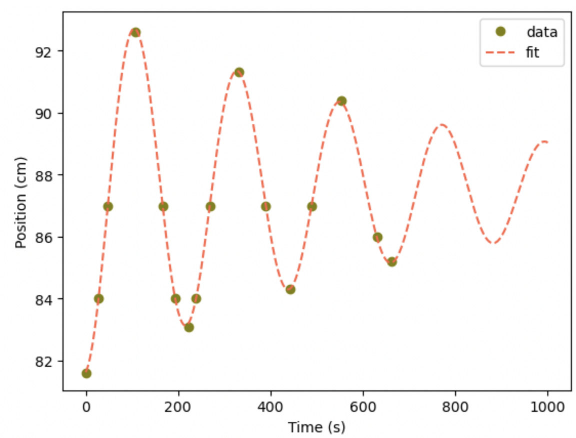

Figure 4: First orientation, trial 2

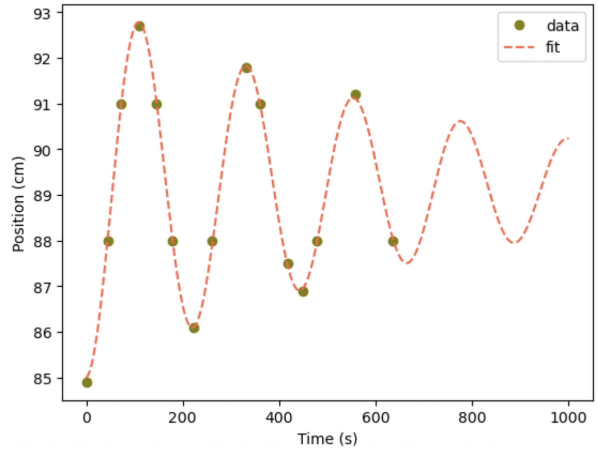

Figure 5: First orientation, trial 3

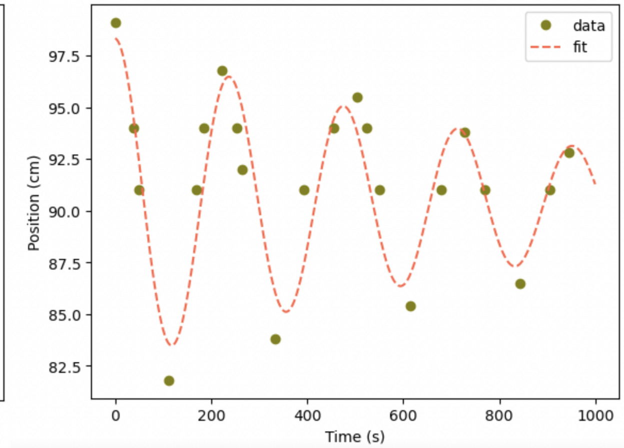

Figure 6: Second orientation, trial 1

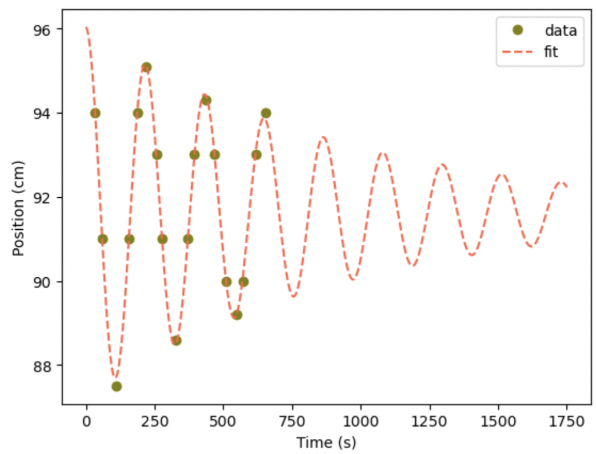

Figure 7: Second orientation, trial 2

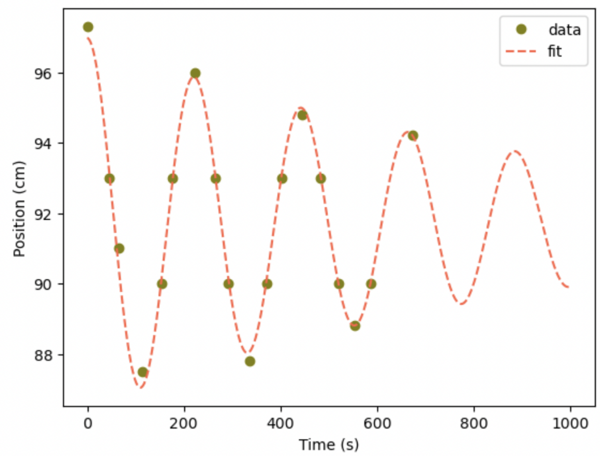

Figure 8: Second orientation, trial 3

Orientation	Trial	Amplitude	λ	ω	ϕ	y_0
1	1	8.7591	$1.2701 \cdot 10^{-3}$	$2.8568 \cdot 10^{-2}$	3.0689	87.3466
1	2	5.9658	$1.3803 \cdot 10^{-3}$	$2.8321 \cdot 10^{-2}$	3.2558	87.5496
1	3	4.1846	$1.3786 \cdot 10^{-3}$	$2.8279 \cdot 10^{-2}$	3.1584	89.1780
2	1	7.9148	$1.1218 \cdot 10^{-3}$	$2.6364 \cdot 10^{-2}$	6.2800	90.4102
2	2	4.4083	$1.0438 \cdot 10^{-3}$	$2.9056 \cdot 10^{-2}$	6.2800	91.6280
2	3	5.2602	$1.0704 \cdot 10^{-3}$	$2.8403 \cdot 10^{-2}$	6.2639	91.7144

Table 1: Initial Parameters

We then used these initial parameters as our starting parameters when fitting using orthogonal distance regression. This fit better incorporates the errors in the x-axis. These plots are shown in **Figures 9-14** and the new final parameters are displayed in **Table 2**.

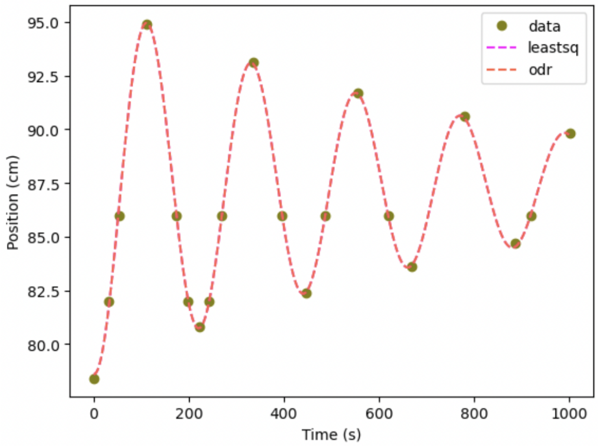

Figure 9: First orientation, trial 1

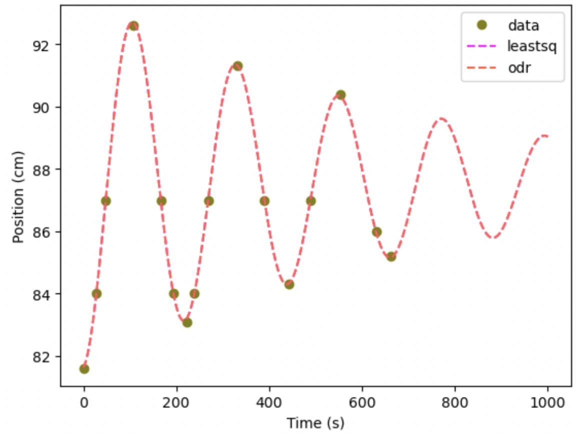

Figure 10: First orientation, trial 2

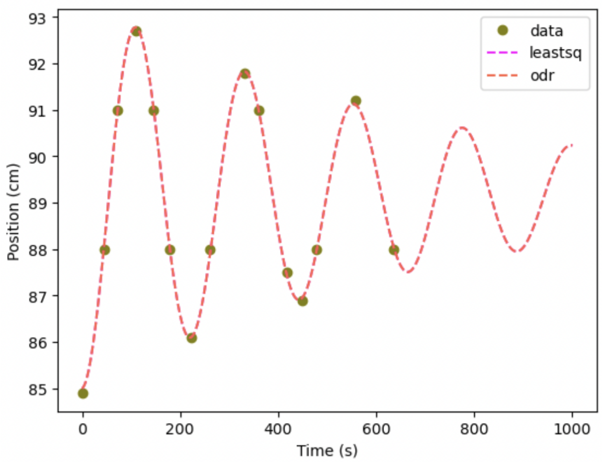

Figure 11: First orientation, trial 3

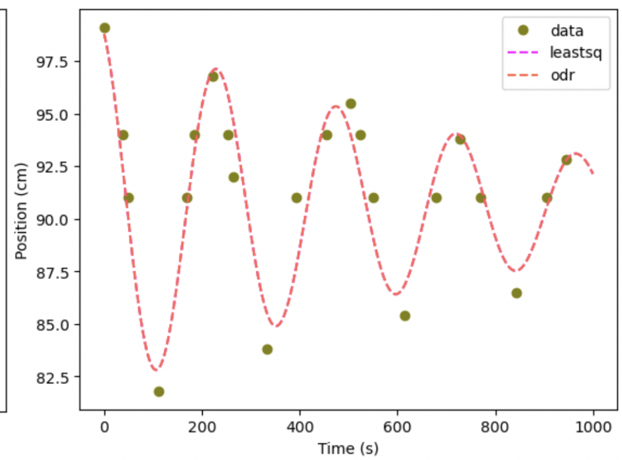

Figure 12: Second orientation, trial 1

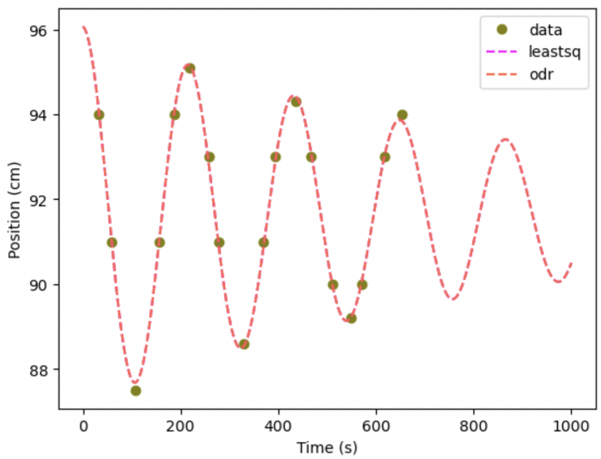

Figure 13: Second orientation, trial 2

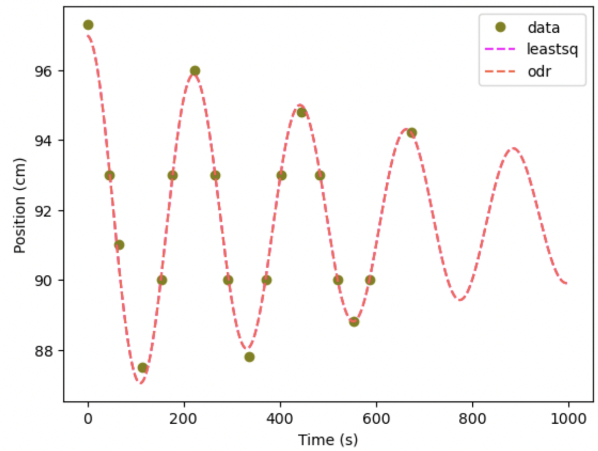

Figure 14: Second orientation, trial 3

Orientation	Trial	Amplitude	λ	ω	ϕ	y_0
1	1	8.7603	$1.2704 \cdot 10^{-3}$	$2.8568 \cdot 10^{-2}$	3.0691	87.3469
1	2	5.9658	$1.3803 \cdot 10^{-3}$	$2.8321 \cdot 10^{-2}$	3.2558	87.5496
1	3	4.1846	$1.3786 \cdot 10^{-3}$	$2.8279 \cdot 10^{-2}$	3.1584	89.1780
2	1	8.8611	$1.2810 \cdot 10^{-3}$	$2.5622 \cdot 10^{-2}$	6.6526	90.5230
2	2	4.4317	$1.0577 \cdot 10^{-3}$	$2.8997 \cdot 10^{-2}$	6.3010	91.6352
2	3	5.2605	$1.0705 \cdot 10^{-3}$	$2.8403 \cdot 10^{-2}$	6.2639	91.7144

Table 2: Final Parameters

The corresponding uncertainties are summarized in **Table 3**.

Orientation	Trial	Amplitude	λ	ω	ϕ	y_0
1	1	$1.4355 \cdot 10^{-1}$	$4.6536 \cdot 10^{-5}$	$6.6984 \cdot 10^{-5}$	$1.9838 \cdot 10^{-2}$	$5.1093 \cdot 10^{-2}$
1	2	$3.9903 \cdot 10^{-2}$	$2.3373 \cdot 10^{-5}$	$3.1291 \cdot 10^{-5}$	$7.9846 \cdot 10^{-3}$	$1.5148 \cdot 10^{-2}$
1	3	$4.9958 \cdot 10^{-2}$	$4.3683 \cdot 10^{-5}$	$6.1497 \cdot 10^{-5}$	$1.4297 \cdot 10^{-2}$	$1.8459 \cdot 10^{-2}$
2	1	1.2107	$3.7812 \cdot 10^{-4}$	$3.6360 \cdot 10^{-4}$	$1.2504 \cdot 10^{-1}$	$3.6408 \cdot 10^{-1}$
2	2	$1.2993 \cdot 10^{-1}$	$8.1814 \cdot 10^{-5}$	$8.2577 \cdot 10^{-5}$	$2.4580 \cdot 10^{-2}$	$3.3458 \cdot 10^{-2}$
2	3	$1.8452 \cdot 10^{-1}$	$1.0994 \cdot 10^{-4}$	$1.3050 \cdot 10^{-4}$	$3.6577 \cdot 10^{-2}$	$6.0197 \cdot 10^{-2}$

Table 3: Standard error of parameters

5 Analysis

5.1 Angle of deflection

The position associated to each equilibrium is represented by y_0 in **Table 2**. The mean equilibrium position for the first orientation is 88.0248 while the mean equilibrium position for the second orientation is 91.2909. The corresponding uncertainties can be calculated using **Equation 6**, from which we obtain

$$\sigma(\bar{y_1}) = \sqrt{(\frac{1}{3})^2\sigma^2(y_{11}) + (\frac{1}{3})^2\sigma^2(y_{12}) + (\frac{1}{3})^2\sigma^2(y_{13})} = \frac{1}{3}\sqrt{\sigma^2(y_{11}) + \sigma^2(y_{12}) + \sigma^2(y_{13})} \quad (9)$$

for orientation 1, and

$$\sigma(\bar{y_2}) = \sqrt{(\frac{1}{3})^2\sigma^2(y_{21}) + (\frac{1}{3})^2\sigma^2(y_{22}) + (\frac{1}{3})^2\sigma^2(y_{23})} = \frac{1}{3}\sqrt{\sigma^2(y_{21}) + \sigma^2(y_{22}) + \sigma^2(y_{23})} \quad (10)$$

for orientation 2. I have introduced a somewhat confusing notation where y_{ab} refers to the y_0 value in **Table 2** that is orientation a and trial b. Furthermore, $\bar{y_c}$ refers to the mean $\bar{y_0}$ value that is orientation c. Thus, for orientation 1,

$$\sigma(\bar{y_1}) = \frac{1}{3}\sqrt{(5.1093 \cdot 10^{-2})^2 + (1.5148 \cdot 10^{-2})^2 + (1.8459 \cdot 10^{-2})^2} = 0.01880 \quad (11)$$

and for orientation 2,

$$\sigma(\bar{y_2}) = \frac{1}{3}\sqrt{(3.6408 \cdot 10^{-1})^2 + (3.3458 \cdot 10^{-2})^2 + (6.0197 \cdot 10^{-2})^2} = 0.1235 \quad (12)$$

Therefore,

$$\bar{y_1} = 88.0248cm \pm 0.01880cm = 0.8802m \pm 0.0001880m \quad (13)$$

$$\bar{y_2} = 91.2909cm \pm 0.1235cm = 0.9129m \pm 0.001235m \quad (14)$$

For our torque calculations, it is necessary to find the deflection angle θ shown in **Figure 15**.

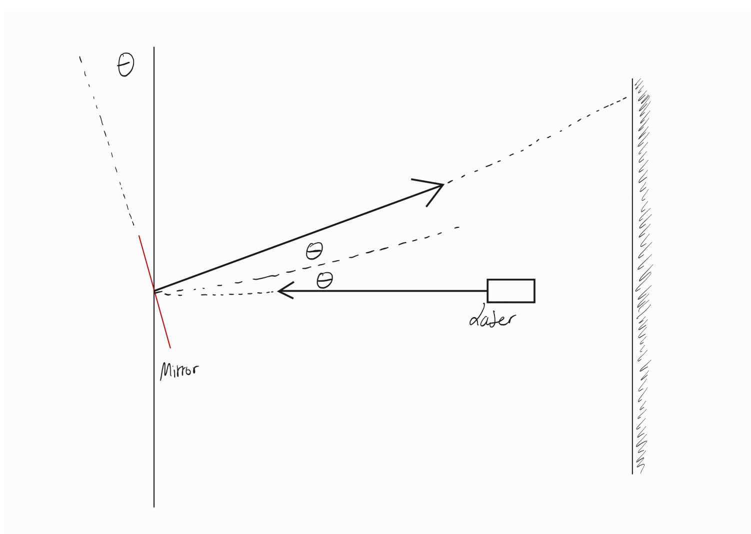

Figure 15: Diagram of angle of deflection

We can compute θ using the the properties of right triangles. First, let us calculate the distance A shown in **Figure 16**.

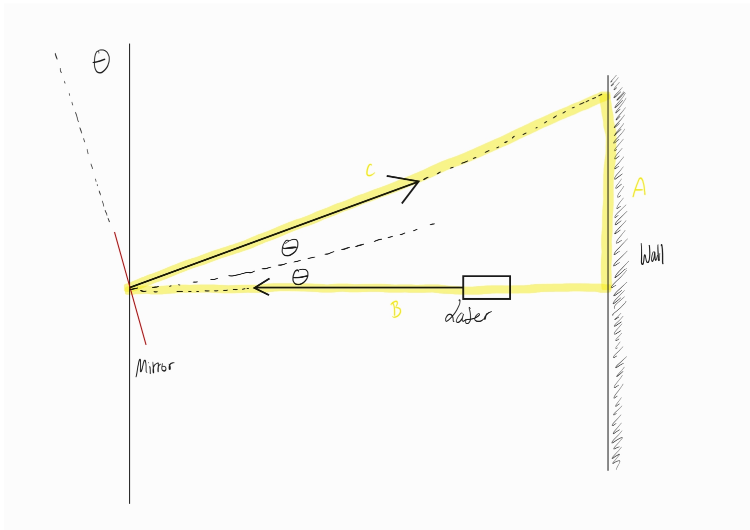

Figure 16: Labeled diagram of angle of deflection

The vertex connecting A and B represents the point on the ruler directly in line with the mirror. We measured this value by hand — it was $82cm \pm 4cm$ or $0.82m \pm 0.04m$. Note that this value need not be the center value between the values $\bar{y}_1$ and $\bar{y}_2$, since the mirror on the balance was not perfectly parallel to the wall. Thus, for the first orientation,

$$A_1 = (0.8802m \pm 0.0001880m) - (0.82m \pm 0.04m) = 0.06020m \pm 4.0188 \cdot 10^{-2}m \tag{15}$$

and for the second orientation,

$$A_2 = (0.9129m \pm 0.001235m) - (0.82m \pm 0.04m) = 0.09290 \pm 4.1235 \cdot 10^{-2}m \tag{16}$$

where the uncertainties come from applying the error propagation rule.

The shortest distance between the mirror and the wall, B, was also measured by hand. The results of our measurements are summarized in the table below.

	Measured Length (m)	Uncertainty (m)
1	6.367	0.002
2	6.364	0.002
3	6.363	0.002
4	6.364	0.002
5	6.359	0.002
6	6.364	0.002

Table 4: Distance from mirror to wall

From the data in **Table 4**, we calculate the mean to be

$$B = 6.364m \pm 8.1650 \cdot 10^{-4}m \tag{17}$$

where the uncertainty comes from applying the error propagation rule. Now we can calculate θ as

$$\theta = \frac{1}{2} tan^{-1}(\frac{A}{B}) \tag{18}$$

where the uncertainty associated to θ is given by **Equation 8**. For the first orientation,

$$\theta_1 = \frac{1}{2}tan^{-1}(\frac{A_1}{B}) = \frac{1}{2}tan^{-1}(\frac{0.06020m \pm 4.0188 \cdot 10^{-2}m}{6.364m \pm 8.1650 \cdot 10^{-4}m}) = 9.4595 \cdot 10^{-3}rad \pm 3.1572 \cdot 10^{-3}rad \quad (19)$$

and for the second orientation,

$$\theta_2 = \frac{1}{2}tan^{-1}(\frac{A_2}{B}) = \frac{1}{2}tan^{-1}(\frac{0.09290m \pm 4.1235 \cdot 10^{-2}m}{6.364m \pm 8.1650 \cdot 10^{-4}m}) = 1.4598 \cdot 10^{-2}rad \pm 3.2390 \cdot 10^{-3}rad \quad (20)$$

5.2 Period

The period of oscillation is related to the angular frequency through the following equation

$$T = \frac{2\pi}{\omega} \quad (21)$$

Thus, we can calculate the periods for each trial using the values for ω in **Table 2**. These results are shown in **Table 5** below.

Orientation	Trial	Period (s)
1	1	219.9346
1	2	221.8570
1	3	222.1823
2	1	245.2262
2	2	216.6840
2	3	221.2142

Table 5: Periods

Since the restoring force is proportional to the displacement of the pendulum, the period is independent of its amplitude. Therefore, we expect the period of each trial to be the same. The mean period using the values from **Table 5** is $224.5164s$.

5.3 Moment of inertia

The moment of inertia I of the pendulum, the aluminum beam plus the two small spheres, can be broken down into three parts: the moment of inertia of the rectangular prism, the moment of inertia of one sphere, and the parallel axis theorem calculation for each sphere. We can calculate the moment of inertia of the beam using the following equation

$$I_{Beam} = \frac{1}{12}m_{Beam}(l^2 + w^2) = \frac{1}{12}(0.00717kg)[(0.149m)^2 + (0.0127m)^2] = 1.3361 \cdot 10^{-5}kg \cdot m^2 \quad (22)$$

where m_{Beam} is the beam's mass, l is its length, and w is its width. If $m_{Beam} = 0.00717kg$, $l =$
The moment of inertia of a single sphere can be calculated using the following equation

$$I_{Sphere} = \frac{2}{5}m_{Sphere}r^2 = \frac{2}{5}(0.0145kg)(0.00675m)^2 = 2.6426 \cdot 10^{-7}kg \cdot m^2 \quad (23)$$

where m_{Sphere} is the sphere's mass and r is its radius. To apply the parallel axis theorem to each sphere, we need to calculate the distance between the center of mass of each sphere and the center of the beam. The moment of inertia of each sphere about the center of the beam can then be calculated using the formula

$$I'_{Sphere} = I_{Sphere} + m_{Sphere}d^2 = 2.6426 \cdot 10^{-7} kg \cdot m^2 + (0.0145kg)(0.0667m)^2 = 6.4773 \cdot 10^{-5} kg \cdot m^2 \tag{24}$$

where d is the distance between the center of the beam and the center of mass of a sphere. Finally, the total moment of inertia of the pendulum about its center can be calculated by summing the moment of inertia of the beam and the moment of inertia of each sphere about the center of the beam

$$I_{Total} = I_{Beam} + 2I'_{Sphere} = 1.3361 \cdot 10^{-5} kg \cdot m^2 + 2(6.4773 \cdot 10^{-5} kg \cdot m^2) = 1.4291 \cdot 10^{-4} kg \cdot m^2 \tag{25}$$

5.4 Torsional constant

If the amplitude of the oscillations is not too large, the period of the oscillator is given by the equation

$$T = 2\pi \sqrt{\frac{I_{Total}}{\beta}} \tag{26}$$

where β is the torsional constant of the tungsten wire. From this, we derive

$$\beta = I_{Total}(\frac{2\pi}{T})^2 = 1.4291 \cdot 10^{-4} kg \cdot m^2 (\frac{2\pi}{224.5164s})^2 = 1.1192 \cdot 10^{-7} \frac{kg \cdot m^2}{s^2} \tag{27}$$

where we used the mean period found in **Section 5.2**.

5.5 Gravitational torque

5.5.1 Nearby large sphere on small sphere

Here we calculate the gravitational torque exerted on each of the two small spheres by each of the two large spheres. First we will calculate the torque on a small sphere due to the nearby large sphere. In the context of **Figure 1**, this could be the torque on A due to $P1$. The equation for gravitational torque is

$$\boldsymbol{\tau}_1 = \boldsymbol{F}_1 \times \boldsymbol{r} \tag{28}$$

where $\boldsymbol{r}$ is the lever arm. Some relevant parameters for our calculations are shown in **Figure 17**.

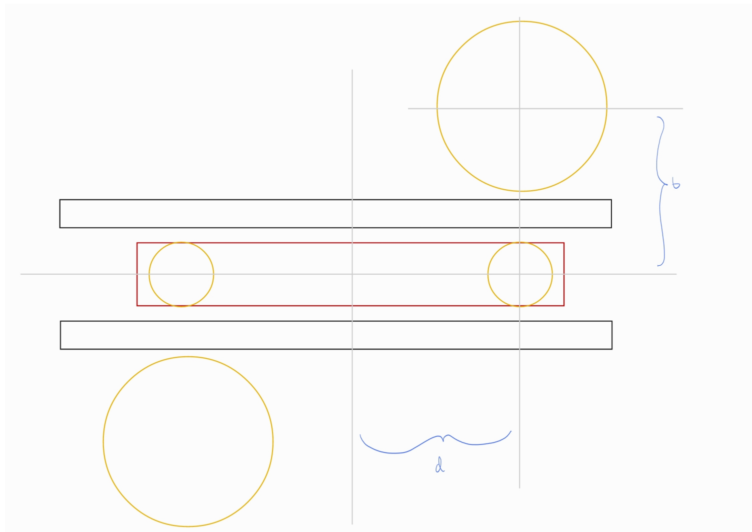

Figure 17: Diagram for torque calculations

As previously mentioned in **Section 1**, the gravitational force between two spheres can be given by Newton's law of gravitation

$$\boldsymbol{F}_1 = G\frac{mM}{b^2}\hat{b} \tag{29}$$

where m is the mass of the small sphere, M is mass the of the large sphere, and b is the distance between the center of masses of the two spheres. Substituting this expression into **Equation 28**, we obtain

$$\boldsymbol{\tau}_1 = G\frac{mM}{b^2}\hat{b} \times \boldsymbol{r} \tag{30}$$

and thus

$$\tau_1 = G\frac{mMr}{b^2} \tag{31}$$

In the context of **Figure 17**, this corresponds to motion in the counterclockwise direction. Finally, substituting the parameter d in for the lever arm, we obtain

$$\tau_1 = G\frac{mMd}{b^2} \tag{32}$$

5.5.2 Farther large sphere on small sphere

Next, we calculate the torque on a small sphere due to the farther large sphere. In the context of **Figure 1**, this could be the torque on A due to $P2$. In this case, the magnitude of the gravitation force between the two spheres is

$$F_2 = G\frac{mM}{4d^2 + b^2} \tag{33}$$

The distance between a small sphere and the farther large sphere is shown in **Figure 18** below, as well as the angle ϕ between the force vector and the lever arm.

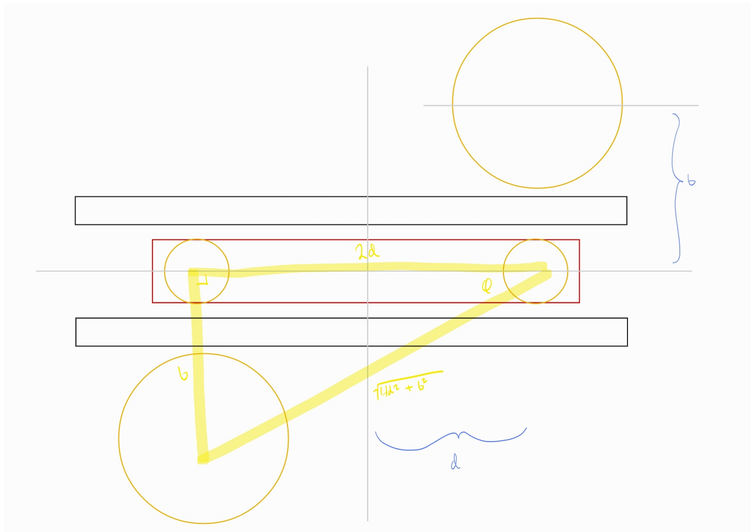

Figure 18: Another diagram for torque calculations

The angle ϕ can be calculated as

$$\phi = tan^{-1}(\frac{b}{2d}) \tag{34}$$

The corresponding torque becomes

$$\boldsymbol{\tau}_2 = \boldsymbol{F}_2 \times \boldsymbol{r} = G\frac{mMr}{4d^2 + b^2} sin[tan^{-1}(\frac{b}{2d})] \tag{35}$$

where τ_2 twists the pendulum in the clockwise direction. Again, substituting the parameter d in for the lever arm, we obtain

$$\tau_2 = G\frac{mMd}{4d^2 + b^2} sin[tan^{-1}(\frac{b}{2d})] \tag{36}$$

5.5.3 Large sphere on beam

If we assume that the beam is a very thin uniform rod, the torque $\boldsymbol{d\tau}$ contributed by an infinitesimally small segment of the beam can be expressed as

$$\boldsymbol{d\tau} = \boldsymbol{F} \times \boldsymbol{x} = Fxsin\theta \tag{37}$$

where F is the magnitude of the force acting on an infinitesimal segment dm that is at a distance x from the center of the beam,

$$F = G\frac{Mdm}{\alpha^2} \tag{38}$$

where α is the distance between the large sphere and the infinitesimal mass dm, which can be expressed as

$$dm = \lambda dx \tag{39}$$

where λ represents the mass per unit length of the beam. Then,

$$d\tau = G\frac{M\lambda dx}{\alpha^2}x sin\theta \tag{40}$$

Thus, the total torque on half of the beam due to the nearby large sphere is

$$\tau_3 = \int_0^{l/2} G\frac{M\lambda dx}{\alpha^2}x sin\theta \tag{41}$$

where we integrate from 0 to $\frac{l}{2}$, half the length of the beam. We can rewrite $sin\theta$ in terms of α through the following relation

$$sin\theta = \frac{b}{\alpha} \tag{42}$$

yielding

$$\tau_3 = \int_0^{l/2} G\frac{M\lambda dx}{\alpha^2}\frac{bx}{\alpha} \tag{43}$$

We can then rewrite α in terms of x through the following relation

$$\alpha = \sqrt{(d-x)^2 + b^2} \tag{44}$$

which yields the integral

$$\tau_3 = \int_0^{l/2} G\frac{M\lambda bx}{[(d-x)^2 + b^2]^2}dx \tag{45}$$

Simplifying this integral, we obtain

$$\tau_3 = GM\lambda b\int_0^{l/2} \frac{x}{[(d-x)^2 + b^2]^2}dx \tag{46}$$

where τ_3 twists the pendulum in the counterclockwise direction, similar to τ_1. Note that τ_3 represents the torque acting on half of the beam due to the nearby sphere, and so just like τ_1 and τ_2, there will be two contributions corresponding to each large sphere. Also note that we are neglecting the torque acting on each half of the beam due to the farther sphere.

5.5.4 Total Torque

To a good approximation, the total torque on the pendulum will be

$$\tau_{tot} = 2\tau_1 - 2\tau_2 + 2\tau_3 \tag{47}$$

where τ_1 and τ_3 cause the pendulum to twist in the counterclockwise direction and τ_2 causes the pendulum to twist in the clockwise direction. Substituting **Equations 32, 36, and 46** into **Equation 47**, we obtain

$$\tau_{tot} = 2[G\frac{mMd}{b^2}] - 2[G\frac{mMd}{4d^2 + b^2}sin[tan^{-1}(\frac{b}{2d})]] + 2[GM\lambda b\int_0^{l/2} \frac{x}{[(d-x)^2 + b^2]^2}dx] \tag{48}$$

Note that **Equation 48** corresponds to the second orientation because it has a net effect of twisting the pendulum in the counterclockwise direction.

5.6 Calculating Gravitational Constant G

Now we can solve for the gravitational constant G in terms of known parameters using **Equation 2**. For the first orientation, **Equation 2** becomes

$$\tau_1 = -\beta(\theta_{cw} - \theta_0) \tag{49}$$

and for the second orientation,

$$\tau_2 = -\beta(\theta_{ccw} - \theta_0) \tag{50}$$

Subtracting these equations, we obtain

$$\tau_2 - \tau_1 = -\beta(\theta_{ccw} - \theta_0) - [-\beta(\theta_{cw} - \theta_0)] = \beta(\theta_{cw} - \theta_{ccw}) \tag{51}$$

which simplifies into

$$\Delta\tau = \beta\Delta\theta \tag{52}$$

where θ_{cw} represents the angle of deflection corresponding the first orientation, and θ_{ccw} represents the angle of deflection corresponding the second. Since τ_1 and τ_2 are equal and opposite, $\Delta\tau$ is simply twice the total torque calculated in **Equation 48**

$$\Delta\tau = 4[G\frac{mMd}{b^2}] - 4[G\frac{mMd}{4d^2+b^2}sin[tan^{-1}(\frac{b}{2d})]] + 4[GM\lambda b\int_0^{l/2}\frac{x}{[(d-x)^2+b^2]^2}dx] \tag{53}$$

It is convenient to write this as

$$\frac{\Delta\tau}{G} = 4[\frac{mMd}{b^2}] - 4[\frac{mMd}{4d^2+b^2}sin[tan^{-1}(\frac{b}{2d})]] + 4[M\lambda b\int_0^{l/2}\frac{x}{[(d-x)^2+b^2]^2}dx] \tag{54}$$

Using $m = 0.0145kg$, $M = 1.508kg$, $d = 0.0667m$, $b = 0.0458m$, $\lambda = 0.04812\frac{kg}{m}$, and $\frac{l}{2} = 0.0745m$ we obtain the value

$$\frac{\Delta\tau}{G} = 8.7896\frac{kg^2}{m} \tag{55}$$

From **Equations 19 and 20**, $\Delta\theta$ becomes

$$\Delta\theta = \theta_{cw} - \theta_{ccw} = [9.4595\cdot10^{-3}rad \pm 3.1572\cdot10^{-3}rad] - [1.4598\cdot10^{-2}rad \pm 3.2390\cdot10^{-3}rad] \tag{56}$$

Yielding

$$\Delta\theta = -5.1385\cdot10^{-3}rad \pm 6.3962\cdot10^{-3}rad \tag{57}$$

Rearranging **Equation 52**, we obtain

$$G = \frac{\beta\Delta\theta}{(\frac{\Delta\tau}{G})} \tag{58}$$

Inputting our value of β from **Equation 27**, our value of $\Delta\theta$ from **Equation 57**, and our value for $\frac{\Delta\tau}{G}$ from **Equation 55**, we obtain the following value for G

$$G = \frac{[1.1192\cdot10^{-7}\frac{kg\cdot m^2}{s^2}][5.1385\cdot10^{-3}rad \pm 6.3962\cdot10^{-3}rad]}{8.7896\frac{kg^2}{m}} = 6.5429\cdot10^{-11}\frac{m^3}{kg\cdot s^2} \tag{59}$$

Using the error propagation rule, we calculate the uncertainty in G to be

$$\sigma(G) = 8.1443 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \tag{60}$$

6 Discussion

The Cavendish experiment is a classic experiment that involves the delicate measurement of the torsional motion induced by gravitational forces between small masses and large spheres. By carefully observing and analyzing the torsion balance's behavior, we were able to calculate the gravitational constant G. Our measured value for G was found to be $6.5429 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$, with an uncertainty of $8.1443 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$. The theoretical value of G, $6.6743 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$, is remarkably close to our estimate, and our uncertainty analysis indicates that it falls within our uncertainty range. The fact that our experimental value falls within the uncertainty range indicates that our measurement is consistent with the accepted value and supports the validity of our experimental setup and data analysis.

However, it is important to acknowledge the sources of uncertainties in our experiment. Several factors can contribute to uncertainties, such as measurement errors, environmental disturbances, and limitations of the experimental setup. These uncertainties should be taken into account when interpreting the results.

One potential source of uncertainty is the measurement of the equilibrium positions. Although we took multiple measurements and averaged the results, there may still be inherent errors in the measurement process. For example, it is possible that we had a tendency to record the time too slowly per position, although one could argue that as long as this error was somewhat consistent, it should not have affected our results since we would just be shifting the x-axis of our plots by a constant value. With that said, improving the measurement technique or using more precise equipment would likely increase the precision of our equilibrium positions.

Furthermore, environmental disturbances can also introduce uncertainties. Variations in temperature can cause expansion or contraction of the apparatus, leading to uncertainty in the distances we measured. Despite implementing shielding measures, air currents may still create additional forces. Electromagnetic interference from the surroundings could also introduce spurious forces. Finally, errors in the calibration of any of the instruments, such as the torsion balance, could also have affected our measurements.

To further improve the accuracy of the experiment, increasing the number of trials and measurements can provide more reliable results and reduce statistical uncertainties. This would involve repeating the experiment multiple times and analyzing a larger dataset.

6.1 Imperfect Initial Alignment of the Torsion Balance

We mentioned in **Section 2** that the experiment was set up so that the long axis of the inner beam is parallel to the face of the transparent plates. One source of error is the imperfect alignment of this inner beam before any torques are applied. Let us assume that the inner beam is displaced by an angle ϵ from the horizontal, as shown in **Figure 19**.

The distance between the large sphere and the small sphere is now

$$distance = \sqrt{[d(1 - cos\epsilon)]^2 + [b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon]^2} \tag{61}$$

Thus, our force calculation in **Section 5.5.1** becomes

$$F = G \frac{mM}{[d(1 - cos\epsilon)]^2 + [b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon]^2} \tag{62}$$

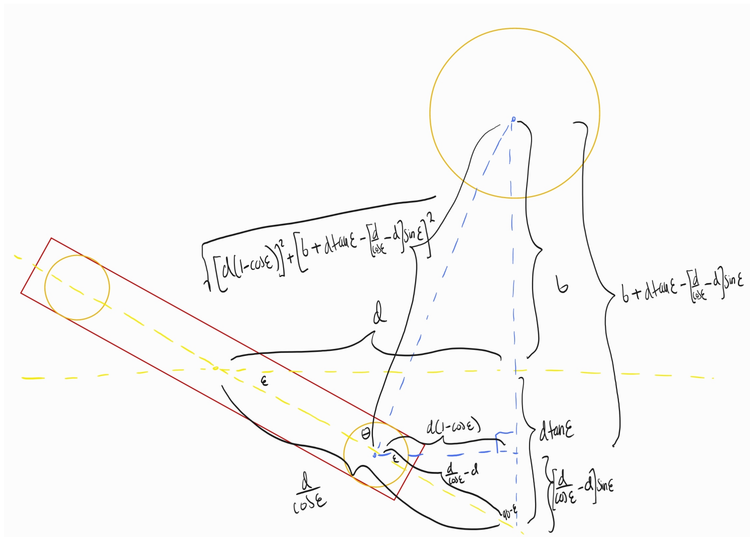

Figure 19: Imperfect Alignment Calculations

and our torque calculation becomes

$$\tau = G\frac{mMr}{[d(1-cos\epsilon)]^2 + [b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon]^2}sin\theta \tag{63}$$

where θ is the angle between the force vector and the lever arm,

$$\theta = 180 - \epsilon - arctan[\frac{b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon}{d(1-cos\epsilon)}] \tag{64}$$

Thus, substituting in d for the lever arm and the expression for θ, we obtain

$$\tau = G\frac{mMd}{[d(1-cos\epsilon)]^2 + [b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon]^2}sin[180 - \epsilon - arctan[\frac{b + dtan\epsilon - (\frac{d}{cos\epsilon} - d)sin\epsilon}{d(1-cos\epsilon)}]] \tag{65}$$

Let's assume a $0.1rad$ misalignment and recalculate G. For $\epsilon = 0.1rad$, we obtain a torque of

$$\tau_1 = 0.6918G \tag{66}$$

which yields

$$\tau_{tot} = 2[0.6918G] - 2[G\frac{mMd}{4d^2+b^2}sin[tan^{-1}(\frac{b}{2d})]] + 2[GM\lambda b\int_0^{l/2}\frac{x}{[(d-x)^2+b^2]^2}dx] \tag{67}$$

Then

$$\frac{\Delta\tau}{G} = 4[0.6918] - 4[\frac{mMd}{4d^2+b^2}sin[tan^{-1}(\frac{b}{2d})]] + 4[M\lambda b\int_0^{l/2}\frac{x}{[(d-x)^2+b^2]^2}dx] \tag{68}$$

yielding

$$\frac{\Delta\tau}{G} = 8.9661\frac{kg^2}{m} \tag{69}$$

and so

$$G = \frac{[1.1192 \cdot 10^{-7} \frac{kg \cdot m^2}{s^2}][5.1385 \cdot 10^{-3} rad]}{8.9661 \frac{kg^2}{m}} = 6.4142 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \tag{70}$$

Thus, an of $0.1 rad$ had the effect of lowering G by $1.287 \cdot 10^{-12}$, indicating that an of $-0.1 rad$ would raise G by this amount, leaving us with a G value of $6.6716 \cdot 10^{-11}$. This is remarkably close to the theoretical value for G, and thus it is very possible that a small misalignment of the beam caused our value to be slightly off.

7 Conclusion

In conclusion, the Cavendish experiment, conducted by Henry Cavendish in 1797, provided crucial insights into the nature of gravity and the measurement of the gravitational constant. Through his innovative and meticulous setup involving two lead spheres and a torsion balance, Cavendish was able to indirectly measure the gravitational force between the spheres and determine the value of the gravitational constant.

The experiment's significance lies in its contribution to our understanding of universal gravitation, confirming Isaac Newton's inverse square law and providing experimental evidence for the existence of gravitational attraction between objects. By accurately measuring the small torsional deflections caused by the gravitational force, Cavendish was able to determine the gravitational constant, which represents the strength of the gravitational interaction between masses.

The Cavendish experiment not only solidified Newton's theory of gravity but also paved the way for future discoveries and developments in the field of physics. Its influence extends to various areas, including cosmology, astrophysics, and even modern technological advancements. The Cavendish experiment stands as a testament to the scientific method and the relentless pursuit of knowledge. It remains a landmark achievement in physics, continually inspiring scientists to delve deeper into the workings of the universe and fostering a greater understanding of gravity's fundamental role in shaping our world.