The Frank-Hertz Experiment

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Contents

1 Introduction

In 1901, Max Planck proposed that an oscillator of frequency ν only emits or absorbs energy in integral multiples of $h\nu$, an idea inspired by the observed frequency spectrum of blackbody radiation. Einstein then suggested that the photoelectric effect is the quantized transfer of energy from the electromagnetic field to an ejected photoelectron in the form of energy packets called photons. Each incident photon delivers its entire energy $h\nu$ to a single electron.

German physicists James Franck and Gustav Hertz began experiments in 1914 where they bombarded gases with electrons. They wanted to see if the kinetic energy from an electron could cause a gas atom to move to a steady state of higher energy, in which case the electrons would lose kinetic energy in a quantized fashion. They discovered that gas atoms do indeed absorb kinetic energy from electrons in quantized amounts, and in a subsequent experiment, they demonstrated that the excited atoms release the absorbed energy in the same amounts.

2 Experimental Setup

We will use an apparatus similar to that of Franck and Hertz – this is shown in **Figure 1**. The primary component is a tube containing three electrodes. The cathode is heated indirectly via a nearby filament to release electrons, which are then accelerated towards the anode by the potential difference V_a. The anode is a mesh, enabling electrons to pass through and arrive at the repeller. The repeller maintains a potential V_r set by a battery such that it is positive relative to the anode. Thus, only electrons with a kinetic energy above eV at the anode will make it as far as the repeller. A picoammeter positioned at the repeller measures the current I generated by the electrons. Since the current values will be extremely small, we protect the tube from external electromagnetic interference by linking the power ground to the metallic walls.

The tube is almost entirely devoid of gas, except for the presence of mercury. When at room temperature, the vapor pressure of mercury is just $1.6 \cdot 10^{-6}$ atm, thus providing a meager amount of vapor for electrons to encounter when traveling from the cathode to the anode. With that said, at 200°C, the vapor pressure rises to 0.023 atm, increasing the likelihood of interactions between electrons and mercury atoms. To regulate the temperature, the tube is placed inside an oven that features an electric heater controlled by a variac.

Increasing V_a increases the current between the cathode and anode. However, when V_a reaches a certain threshold, inelastic collisions between the electrons and mercury atoms will occur near the anode. During such collisions, an electron loses most of its kinetic energy to the atom, causing the atom to jump to its first excited energy state. The electrons involved in collisions will not be able to overcome the retarding potential V_r, and so the current at the repeller decreases. As V_a is increased even more, the location of the initial inelastic collisions shifts closer to the cathode. These electrons will have enough time to be re-accelerated and reach the repeller, leading to a resurgence in the measured current.

At some point, V_a will be large enough for an electron to experience two successive inelastic collisions. The electron will excite an atom located halfway between the cathode and the anode, then lose all its energy, before being re-accelerated to excite another atom near the anode, ultimately rendering the electron unable to reach the repeller. This trend continues as the accelerating voltage is increased, leading to scenarios with higher multiplicities of inelastic collisions. The result is a cyclic pattern in the repeller current with increasing accelerating voltage.

3 Analysis

An accelerating voltage V' is given by the equation

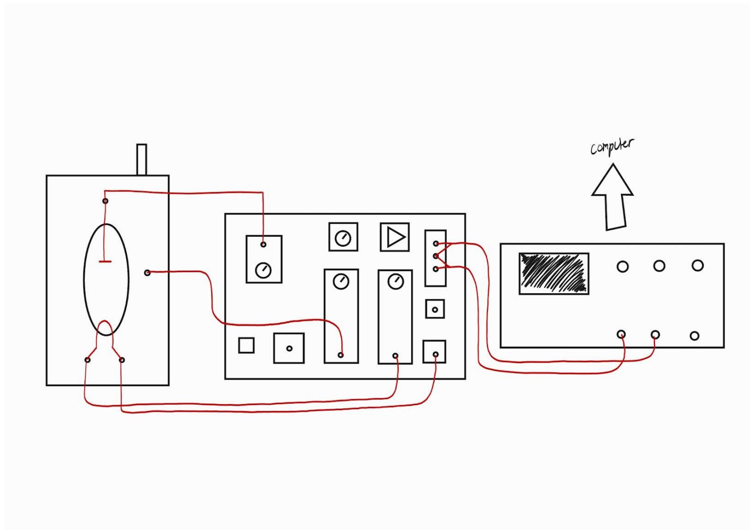

Figure 1: Experimental setup

$$V' = V \cdot S \tag{1}$$

where V is a voltage read off of the oscilloscope and S is the scale factor associated with the x-axis of the oscilloscope.

If the variables **x** are independent, the error propogation law becomes

$$\sigma^2(y_i) = \sum_{j=1}^{n} (\frac{\partial y_i}{\partial x_j})^2 \sigma^2(x_j) \tag{2}$$

Since V and S are independent, we can apply **Equation 2** to **Equation 1**, yielding

$$\sigma^2(V') = S^2\sigma^2(\mathrm{V}) + V^2\sigma^2(\mathrm{S}), \tag{3}$$

or

$$\sigma(V') = \sqrt{S^2\sigma^2(\mathrm{V}) + V^2\sigma^2(\mathrm{S})}, \tag{4}$$

the uncertainty of V'. In order to solve **Equation 4**, we must identify S, $\sigma^2(V)$, and $\sigma^2(S)$. Let us start with our scale factor S, which is defined as

$$S \equiv \frac{\text{true scale}}{\text{oscilloscope scale}}. \tag{5}$$

where 'oscilloscope scale' refers to the full x-axis of the oscilloscope and 'true scale' refers to the corresponding accelerating voltage. In this experiment, *a 5V oscilloscope scale corresponded to a 48.9V accelerating voltage, with an uncertainty of 0.2V*. With that said, we will also use data with a 2V oscilloscope scale, even though we are ignorant of the corresponding true scale. We will use the scale factors

$$S_{5V} = \frac{48.9}{5} = 9.78 \tag{6}$$

$$S_{2V} = \frac{48.9 \cdot \frac{2}{5}}{2} = 9.78 \tag{7}$$

where the true scale of S_{2V} is adjusted proportionally, so that $S_{5V} = S_{2V} = 9.78$. Then, according to **Equation 1**,

$$\sigma^2(S) = [\frac{1}{\text{oscilloscope scale}}]^2\sigma^2(\text{true scale}) + [\frac{\text{true scale}}{\text{oscilloscope scale}}]^2\sigma^2(\text{oscilloscope scale}) \tag{8}$$

or,

$$\sigma^2(S) = [\frac{1}{\text{oscilloscope scale}}]^2\sigma^2(\text{true scale}), \tag{9}$$

since there is no uncertainty associated with the oscilloscope scale. It follows that

$$\sigma^2(S_{5V}) = [\frac{1}{5}]^2[0.2]^2 = 0.0016 \tag{10}$$

or,

$$\sigma(S_{5V}) = \sqrt{\sigma^2(S_{5V})} = 0.04 \tag{11}$$

where $\sigma(S_{5V})$ refers to the uncertainty of the scale factor for an oscilloscope scale of 5V. In regard to the 2V oscilloscope scale, we will add in quadrature an additional 5% uncertainty as we are not privy to the corresponding true scale, yielding

$$\sigma^2(S_{2V}) = [\frac{1}{2}]^2[0.2 \cdot \frac{2}{5}]^2 + [0.05]^2 = 0.0016 + 0.0025 = 0.0041 \tag{12}$$

or,

$$\sigma(S_{2V}) = \sqrt{\sigma^2(S_{2V})} = 0.06403 \tag{13}$$

It follows that

$$S_{5V} = 9.78 \pm 0.04000 \tag{14}$$

$$S_{2V} = 9.78 \pm 0.06403 \tag{15}$$

Finally, the uncertainty associated with V, $\sigma(V)$, was chosen by hand:

$$\sigma_{5V}(V) = 0.025V \tag{16}$$

$$\sigma_{2V}(V) = 0.01V \tag{17}$$

We can now use **Equation 1** to calculate the accelerating voltages and their corresponding uncertainties for each peak in the oscilloscope graphs.

4 Results

In order to obtain the periodicity of the current variations, we will plot *peak number* versus *voltage* – as shown in **Figure 2** – and then identify the slope.

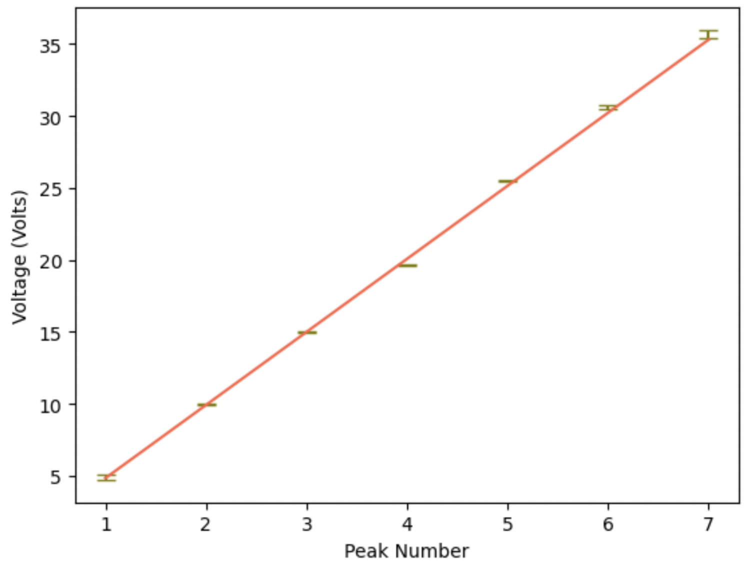

Figure 2: Plot of *peak number* vs *voltage*

4.1 Error Bars

For any given peak number, the corresponding uncertainty is obtained by applying **Equation 2** to the mean of all the voltage values for that peak. The equation for the mean is

$$\bar{x} = \frac{x_1 + x_2 + ...}{n} \tag{18}$$

where **x** are the voltage values associated with a given peak, and n is the number of data points for that peak. Then, according to the error propagation rule,

$$\sigma(\bar{x}) = \sqrt{(\frac{1}{n})^2\sigma^2(x_1) + (\frac{1}{n})^2\sigma^2(x_2) + ...} = \frac{1}{n}\sqrt{\sigma^2(x_1) + \sigma^2(x_2) + ...} \tag{19}$$

Thus, the uncertainty of the mean is just the individual uncertainties added in quadrature, multiplied by a factor of $\frac{1}{n}$. The following code reflects this calculation:

```
# Determine uncertainty of mean
def mean_uncertainty(lst):
    sum_of_squares = 0
    for num in lst:
        sum_of_squares += num**2
    return (1/len(lst))*math.sqrt(sum_of_squares)

# Create lists for means and uncertainties
avgs = []
uncrtnts = []
for i in x:
    avgs.append(np.mean(ps[i-1]))
    uncrtnts.append(mean_uncertainty(pus[i-1].tolist()))

# Plot the error bars for each peak
```

```
for i in range(len(x)):
    plt.errorbar(x[i], avgs[i], yerr=uncrtnts[i], fmt='none', color = 'olive')
```

4.2 Weighted Linear Fit

Since the uncertainty of each mean voltage is different, we must use a weighted linear regression. As shown in the code below, we use the *statsmodels* library where weights for each observation are calculated as the inverse of the square of the uncertainties of the observations.

```
# Calculate the weighted linear fit
w = 1 / np.array(uncrtnts)**2
X = sm.add_constant(x)
model = sm.WLS(avgs, X, weights=w)
result = model.fit()
slope = result.params[1]
intercept = result.params[0]
```

The code first calculates the weights based on the uncertainties. Then, a weighted linear regression model is created using the input data, response variable, and weights. The model is fit to the data, and the estimated slope and intercept of the linear regression line are extracted from the results. Next, we calculate the errors in the slope and intercept of our weighted fit. The last two lines are formulas based on the theory of linear regression and give the standard errors of the slope and intercept.

```
# Calculate the errors in slope and intercept
sum_w = np.sum(w)
sum_wx2 = np.sum(w * x**2)
delta = sum_w * sum_wx2 - (np.sum(w * x))**2
slope_err = np.sqrt(sum_w / delta)
intercept_err = np.sqrt(sum_wx2 / delta)
```

From our fit, we obtained a slope of 5.08 ± 0.02 and an intercept of -0.25 ± 0.07.

5 Discussion

In this experiment, we aimed to determine the energy of the first excited state of a mercury atom using the method developed by Franck and Hertz. The slope of the plot shown in **Figure 2** represents the periodicity of the current variations, which is also the first excited state of the mercury atom. Our measurement yielded a value of 5.08 eV with an uncertainty of 0.02 eV. This value is in good agreement with the theoretical prediction of 4.9 eV. Although this value is not within the uncertainty range for our slope, it is exceptionally close.

The intercept represents the contact potential generated by the two different metals used for the cathode and anode, which we measured to be -0.25 volts with an uncertainty of 0.07 volts. This result is consistent with our expectations, as a negative contact potential is necessary to prevent electrons from reaching the anode until they have acquired enough kinetic energy to overcome the potential barrier.

It is important to note that our measurements were subject to various sources of error. One potential source of error is the accuracy of our voltage measurements, which may have been affected by factors such as electrical noise or calibration errors. Additionally, the temperature of the tube may have varied slightly during the experiment, potentially affecting the observed energy levels.

Overall, our results provide a useful confirmation of the theory behind the Frank-Hertz experiment, and demonstrate the potential of this technique for measuring the energy levels of atoms.

Future studies could aim to further refine our measurements and reduce sources of error, for example by using more precise instruments or controlling for temperature fluctuations more effectively.

6 Conclusion

The Frank-Hertz experiment provides important insights into the nature of energy quantization and atomic excitation. By measuring the current through a tube containing mercury atoms under different accelerating voltages, we observed a series of distinct step-like increases in the current, each corresponding to the excitation of mercury atoms to higher energy levels. These findings confirmed the theoretical predictions of quantized energy levels within atoms, and provided a compelling demonstration of the validity of quantum mechanics. Furthermore, the experiment demonstrated the importance of careful experimental design and control, and highlighted the importance of shielding the apparatus from external electromagnetic interference. Overall, the Frank-Hertz experiment has been a crucial component in the development of our understanding of quantum mechanics and atomic physics, and remains a classic example of experimental physics to this day.