PHY64 Lab 4

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Contents

1 Introduction

1.1 Quantization of Light: Historical Development

Quantization of light was first proposed by Max Planck in 1900 as a solution to the "ultraviolet catastrophe", or the prediction that energy radiated by a blackbody should increased continuously with frequency. Naturally, this cannot be true. To resolve this disagreement between classical electromagnetism and experimental evidence (and also common sense), Max Planck postulated that the energy spectrum of radiation is quantized rather than continuous.

The idea that light is quantized was further validated by Einstein's 1905 model of the photoelectric effect. Here, Einstein used a light beam incident on the surface of a metal to generate an electron current. He found that the energy of the emitted electrons was directly proportional to the frequency of the light in the incident beam (rather than the intensity of the beam).

1.2 Compton Effect

This theory was again validated by Arthur Compton's studies on the interaction between high frequency light and matter.

Compton scattering is a process in which a high-energy photon (such as an X-ray or a gamma ray) scatters off of an electron. In this collision, the photon path is deflected, and some of its energy is transferred to an electron. The scattered photon has less energy, and therefore a reduced frequency (since energy of a photon is given by $E = hf$). The amount of energy lost by the photon is a function of the angle at which it is scattered (as well as the energy of the electron that it interacted with).

1.3 The Compton Equation

We use Einstein's relativistic equation to find the energy of the incident photon:

$$E^2 = p^2c^2 + m^2c^4 \tag{1}$$

For massless particles such as photons, this equation reduces to:

$$E = pc \tag{2}$$

Following the collision, the energy of the scattered photon can be represented as:

$$E' = \frac{E}{1 + \frac{E}{mc^2}(1 - cos\theta)} \tag{3}$$

2 Calibration

The calibration data (taken at a detector angle of zero degrees) is displayed below. There are two useful values that we can pull from this data. A discussion of the fit method and physical interpretation is included as well.

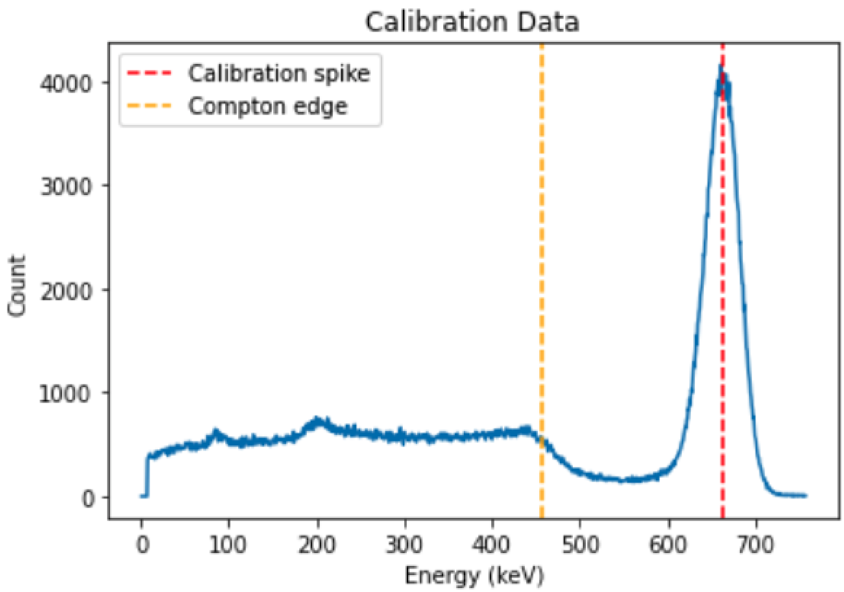


Figure 1: Calibration data with fits

2.1 Photopeak

The primary feature shown here is the 662keV emission of the Cs-137 source. Since the detector is placed at zero degrees relative to the source, the emitted photons shown here have not undergone scattering. Their detected energy therefore matches their initial energy. To determine the energy scale of the plot, we make a fit to this photopeak, and assign its central frequency 662keV. We then assume that the x-axis of the plot scales linearly.

To fit the photopeak data, we manually isolated its surrounding data-window. To do this, we simply created a truncated version of the csv data in the region surrounding the peak. Next, we fit the peak to a gaussian curve. The results of this fit are shown below. Finally, we stored the central bin number for energy calibration.

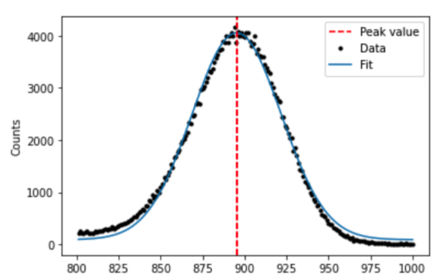


Figure 2: Calibration photopeak with fits

2.2 Compton Edge

Compton scattering can also occur in the detector module. When this occurs, the incident photons recoil off of an electron and carry away some of the energy. The energy carried away by this scattered photon is once again a function of its scattering angle. To find the maximum kinetic energy transferred to the electron, we need to minimize the energy of the scattered photon. We know that the scattered photon's energy is given by:

$$E' = \frac{E}{1 + \frac{E}{mc^2}(1 - cos\theta)} \tag{4}$$

Here, we can see that the photon's energy is minimized when the scattering angle $\theta = 180\,\text{deg}$. Substituting this into the equation yields:

$$E'_{min} = \frac{E}{1 + 2\frac{E}{mc^2}} \tag{5}$$

The maximum energy transferred to the electron is therefore

$$E - E'_{min} = T'_{max} = E - E\left(\frac{1}{1 + 2\frac{E}{mc^2}}\right) \tag{6}$$

$$T'_{max} = E\left(1 - \frac{1}{1 + 2\frac{E}{mc^2}}\right) \tag{7}$$

$$T'_{max} = E\left(1 - \frac{2\frac{E}{mc^2}}{1 + 2\frac{E}{mc^2}}\right) \tag{8}$$

$$T'_{max} = \frac{2E^2}{2E + mc^2} \tag{9}$$

At the calibration point, the energy of the photons hitting the detector should be 662keV. Substituting this value into the equation, along with the mass of an electron yields $T'_{max} = 477.910eV$. This maximum transferred energy gives us the "Compton edge" feature. This is the maximum energy that we can detect from scattered electrons. Energies smaller than this can be In practice, this edge is not a sharp cutoff. Energies above this value may be caused by multiple scattering. The continuum of energies recorded below this value are due to scattering at off-angles, where less energy is transferred from the photon to the electron.

Although there should be a sharp cutoff at the location of the Compton edge, the actual data shows a slightly blurred peak. This is likely due to multiple scatterings in the detector; electrons with a nonzero initial velocity can scatter to have a detection velocity higher than the value calculated above.

To fit this cusp-like data we needed to device a novel fit method. This was difficult due to the fact that we do not have a mathematical model for the distribution. To fit the data, we first manually selected a window of the datafile surrounding the edge. Next, we wrote a python script that parses through the data in the window. As it moves through the data, it calculates a "current average" value of a group of 3 data points. If this average value falls below 90% of the way between of the average baseline value before and after the edge, then it identifies this as the compton edge. A plot of this fit is shown below:

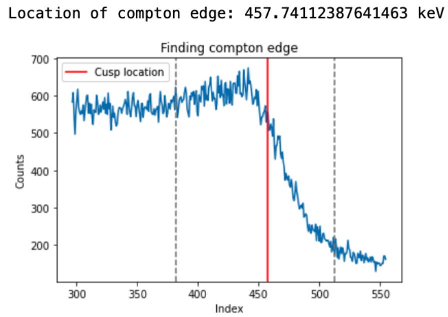


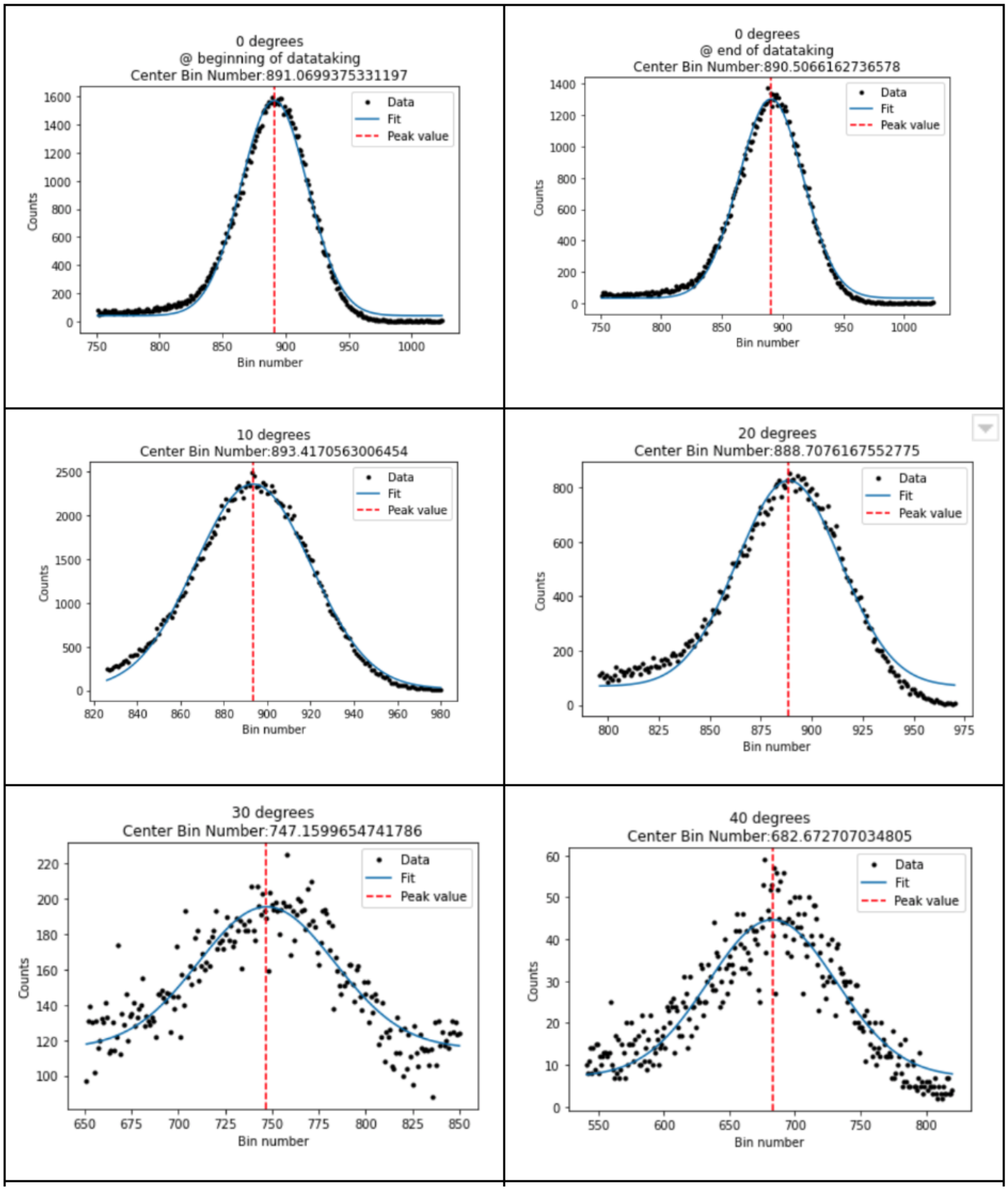
Figure 3: Compton edge with fits

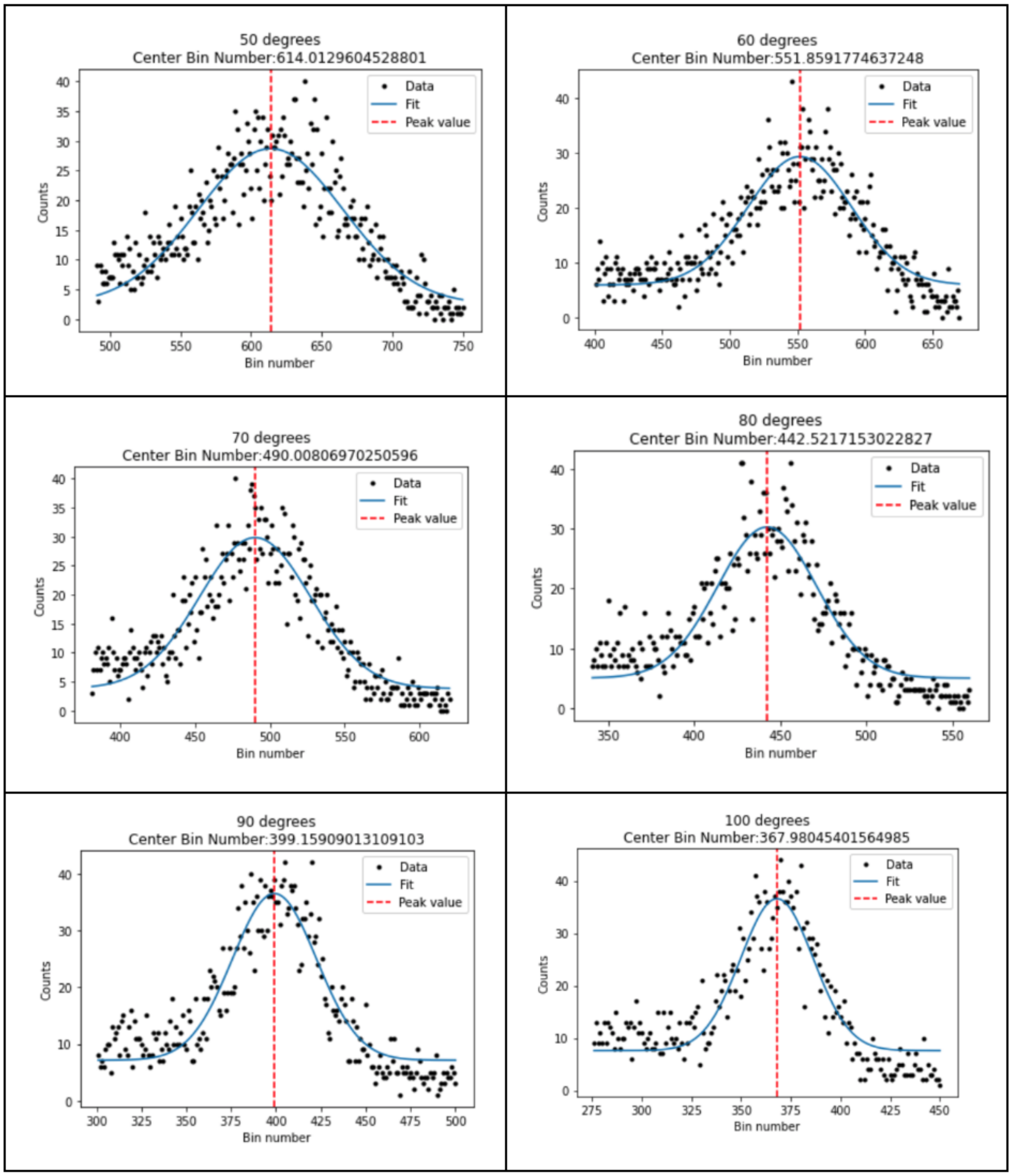
Here, the grey lines display the average value before and after the compton edge. The red line shows the point where the average value of the 3 surrounding datapoints is at the 90% point between the y-values of the two grey lines.

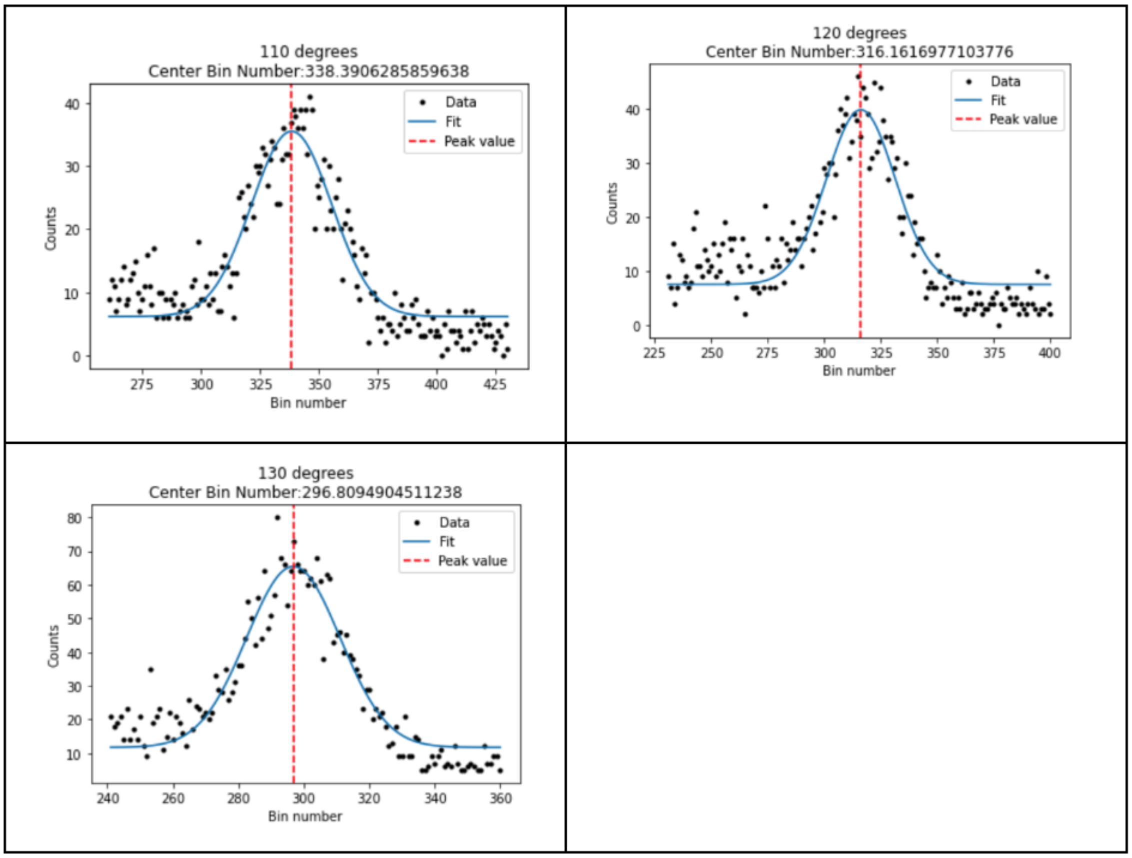
3 Analysis

Following the photopeak procedure, we fit all of our data to gaussian curves. The results of this fitting is shown on the following pages:

3.1 Fits to the data




50 degrees
Center Bin Number:614.0129604528801
60 degrees
Center Bin Number:551.8591774637248
70 degrees
Center Bin Number:490.00806970250596
80 degrees
Center Bin Number:442.5217153022827
90 degrees
Center Bin Number:399.15909013109103
100 degrees
Center Bin Number:367.98045401564985
Data
Fit
Peak value
Counts
Bin number



3.2 Calculating Electron Mass

In this case, the the bin number of our fit is converted to a energy ratio using the following procedure:

$$E_{peak} = n_{peak} \cdot \frac{E_{calibration}}{n_{calibration}} \tag{10}$$

$$Energy\ ratio\ = \frac{E_{calibration}}{E_{peak}} \tag{11}$$

Here, the “n” terms represent the bin-numbers for the measured and calibration peaks. The first equation functions to convert the bin number of the photopeak to an energy value. The second equation is used to convert the energy value to a ratio to be plotted. Combining these equations, we find:

$$Energy\ ratio\ = \frac{n_{calibration}}{n_{peak}} \tag{12}$$

The energy ratio directly maps the fits to bin numbers. This makes it easy to propagate uncertainties in our fits. To do this, we start with the propagation of uncertainty formula:

$$\sigma^2(y_i) = \sum_{j=1}^{n} (\frac{\partial y_i}{\partial x_j})^2 \sigma^2(x_j) \tag{13}$$

In our case, we expect uncertainties both from the fit to each angle's peak value, as well as the fit to the the calibration photopeak (used to set the energy scale). First, we calculate the term due to peak fit errors. This propagates as:

$$\left(\frac{-n_{calibration}}{n_{peak}^2}\right)^2\left(\sigma(n_{peak})\right)^2 \tag{14}$$

Next, we consider the error in the energy calibration scale. We calibrated the location of the photopeak using the fit of the dataset taken with the sensor at the 0-degree mark. We took two datasets with the sensor in this location: one at the beginning, and once at the end of the lab. Using the first dataset, the calibration peak was found to be centered at bin number 891.069. Using the final dataset, the calibration peak was found to be centered at bin number 891.070. The uncertainties are ±0.0484 and ±0.049546 respectively.

We will use the average of these two values for the calibration peak bin number. To find an uncertainty in this measurement, we propagate the errors on the two fits to their average value. An average of two numbers is calculated using the equation:

$$n_{calibration} = \frac{1}{2}(n_1 + n_2) \tag{15}$$

The uncertainty in this value is therefore:

$$\sigma(n_{calibration}) = \sqrt{\left(\frac{\sigma(n_1)}{2}\right)^2 + \left(\frac{\sigma(n_2)}{2}\right)^2} = 0.03463 \tag{16}$$

Next, we propagate this uncertainty to the energy ratio formula from above. Its contribution to the uncertainty in energy ratio terms is given by the equation:

$$\left(\frac{1}{E_{peak}}\right)^2\left(\sigma(n_{calibration})\right)^2 \tag{17}$$

The total uncertainty in each energy ratio is therefore given by equation the two equations added in quadrature:

$$\sigma_{energy\_ratio} = \sqrt{\left(\frac{-n_{calibration}}{n_{peak}^2}\right)^2\left(\sigma(n_{peak})\right)^2 + \left(\frac{1}{E_{peak}}\right)^2\left(\sigma(n_{calibration})\right)^2} \tag{18}$$

We use this formula to calculate error bars for each of the energy ratio points. In doing so, we find an uncertainty that increases across the dataset, which makes sense given the decreasing quality of fit accuracy for the smaller photopeaks.

3.3 Uncertainty in Angle

There is also an uncertainty associated with each angle recorded in our dataset. For the sake of consistency, we had one lab member (Toki) set all the angle positions. While setting angle, we approximated uncertainty in this measurement to be approximately ±1°. We are interested in seeing how this error propagates through to the equation for our x-coordinates:

$$x = 1 - cos(\theta) \tag{19}$$

From the propagation of error formula, we find that

$$\sigma_x = \sqrt{\left[sin(\theta)\right]^2\left[\sigma(\theta)\right]^2} \tag{20}$$

Where as stated above, $\sigma(\theta) = 0.0174$ radians. From this equation, we find the following x errors:

```
[0.0,
 0.0030214782914045877,
 0.005951150493866636,
 0.008699999999999998,
 0.011184504408545782,
 0.013329173310270217,
 0.01506884202584923,
 0.016350651601674804,
 0.01713565490241242,
 0.0174,
 0.01713565490241242,
 0.016350651601674804,
 0.015068842025849235,
 0.013329173310270219]
```

Figure 4: Uncertainty in x values on energy ratio plot

Although error bars for these measurements are plotted, they are not immediately visible on the plot, as they are quite small compared to the scale. This is acceptible, and confirms that the major source of error in our experiment is from the distribution of events rather than human measurement error.

3.3.1 Uncertainty in Electron Mass

The python package `scipy.odr` is used to account for errors in both x and y while fitting the data. This package uses the method of Orthogonal Distance Regression to optimize the linear fit.

In this method, we fit the plot to a linear model with a fixed y-intercept (=1). By definition, the energy ratio E_{cal}/E should be equal to 1 at $E = E_{cal}$, which is found at a 0 degree scattering angle (correlating to an x-value of $1 - cos(\theta) = 0$. Fixing this value and fitting to slope alone ensured that our analysis was logically self-consistent. This setup fit yielded a slope of 1.22720391 with an uncertainty of ± 0.02284264. This python function also provided us with a covariance of 0.00594871 for the fit, and a residual variance of 0.08771.

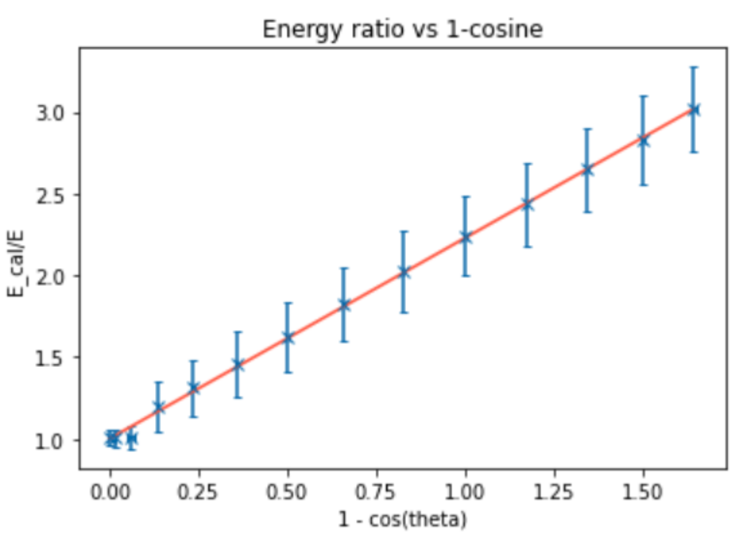


Figure 5: Energy Ratio plotted as a function of 1-cos(theta)

The expected relation between these two quantities is described by the equation:

$$E' = \frac{E}{1 + E(1 - cos(\theta)/mc^2} \tag{21}$$

(as given by equation 4.1 in the lab manual, and discussed in the background section). Rearranging the terms in this equation, we can make this more explicitely map to the plot shown above:

$$\frac{E}{E'} = 1 + \frac{E}{mc^2}(1 - cos(\theta)) \tag{22}$$

We compare this to our line of best fit:

$$y = (1.22742083)x + 1 \tag{23}$$

From this comparison, we set the slope quantities equal:

$$\frac{E}{mc^2} = 1.22742083 \tag{24}$$

And substituting in the known calibration energy of $E = 0.662MeV$, we find:

$$\frac{0.662\ MeV}{mc^2} = 1.22742083 \tag{25}$$

$$mc^2 = 0.539342MeV \tag{26}$$

The uncertainty on this value can be found once again by propagation of the fit uncertainty. We note that previous fit uncertainties are accounted for in this propagation, as they were used to inform the choice of slope in the ODR. They are not explicitly included in this formula, but their agreement to the expected electron mass contributes to the uncertainty of the final fit result.

$$\sigma(mc^2) = \sqrt{\left(\frac{-0.662}{slope^2}\right)^2 \cdot \left(\sigma(slope)\right)^2} \tag{27}$$

Plugging in the fit uncertainty of ± 0.02284264 in our slope, we find:

$$\sigma(mc^2) = \pm 0.0100373 \tag{28}$$

This gives us a final answer of:

$$mc^2 = 0.539342 \pm 0.0100373MeV \tag{29}$$

4 Discussion

Finally, we compare this value to the expected value of the electron mass $0.51099MeV$. We note that this value falls slightly out of our range of predicted values. This means that there is likely a source of unaccounted for error in our measurement. A likely culprit is our assumption that the energy of the Osprey data scales linearly with bin number. In the compton edge fits from above, we found our measure of the energy cutoff to be higher than that of the expected value. This may indicate that the energy scale of the Osprey unit does not have a perfect linear mapping to bin number. Another possible factor is in measurements of angle. There were some minor disagreements on interpretation of viewing angle of the two-arm spectrometer in the lab. Systematic scaling of the angle could have skewed the non-calibration data. This cannot fully explain the discrepancy in the data and the experimental result, because as shown above, uncertainties in the angle translate do not have a major effect when propagated to the final data. Nevertheless, it could play a minor role in explaining the discrepancy. Finally, the method of data-fitting may play a role in the uncertainty. When we do not constrain the y-intercept of the linear fit, we find an expected electron mass of $0.5326 \pm 0.010105 MeV$. Although still incorrect, this is a step in the right direction. This fit to the data found better agreement at lower energy ratios. It was found to have a y-intercept of 0.985. An image of this fit is shown below for reference.

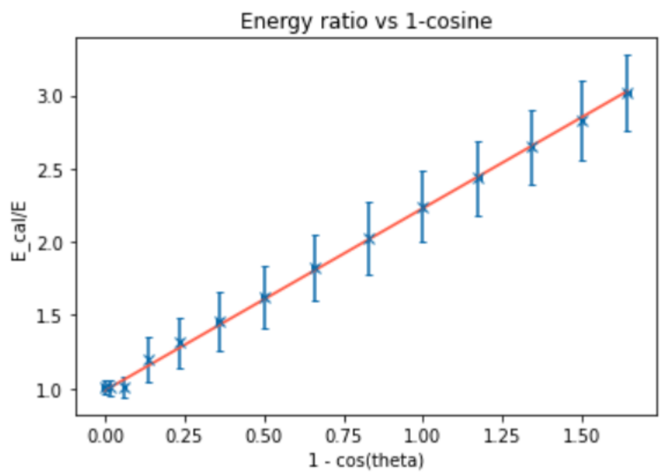


Figure 6: Energy Ratio plotted as a function of 1-cos(theta), with y intercept as a free parameter to the fit

In all, we used this setup to find an effective scatterer mass of 0.539 MeV. We note that this mass is quite close to that of the electron. It does not perfectly agree with the electron mass, but sources of additional uncertainty may explain this. Our data fits cleanly and has seemingly reasonable uncertainty values, so we will accept this as a successful analysis.